Excerpt from PhD Thesis, Andy Blunden. 1971 (p. 75-88)

(B) Investigation of Natural Loading Processes and the Construction of Mathematical Models

Many natural loading processes which exert a load on a structure have the nature of random fluctuations of a quantity, such as displacement or force, with time. That is to say, they are stochastic processes.

Generally speaking, at the present stage of the development of structural analysis it is possible to analyse the effect of any of the prescribed forms which a loading action may take. That is, if we could define a region a space within which the disposition of material was known, and the properties of the material known, and in addition we define all the forces or motions applied at the boundary to this space, then, as a rule, we can satisfactorily predict the motion the material within this space which will ensue.

However, the question awaiting solution at this moment is: how to represent this load in such a way as to be able to lay emphasis on the more frequent or likely members of the set of all possible loading histories, or at least upon the more likely of those loads which will be associated with failure of a structure. Thus, we require some criterion for arranging all the possible loads in order of likelihood of occurrence.

Clearly, the basis for this criterion lies in the study of all previous manifestations of the process and all the physical conditions surrounding these occurrences, together with all the data relating to future events with which we way be concerned. Such considerations fall broadly into two classes.

Firstly, we have the set of all records of the process, that is, time-histories of the fluctuations of the relevant quantity, taken in isolation from the physical (or geographical, historic, etc) data appertaining to them. In particular, records which relate to similar conditions to those with which we may be concerned with in a particular study. For instance, records made at the same, nearby or similar locations, and for some phenomena. perhaps at the same time of year, and so on.

Secondly, we have the whole body of natural science and in particular those fields related to the process concerned, together with knowledge and study of the particular conditions under which the process has, and may exert itself. For instance, in the study of earthquakes, as a structural load, the researcher may draw upon seismology, geology, wave mechanics, soil mechanics and many other branches of physics even if only by way of analogy.

It is the opinion of the author that any technique which is based solely on either one of these two elements of our knowledge of a process is necessarily of limited application or potential.

Before discussing the particular limitations of these two lines, based on one or the other of these categories of our knowledge of a process, which appear in the study of random phenomena, we must mention that approach which altogether denies the necessity of having recourse to existing knowledge of the natural process. This is the purely idealist line. It can exert a surprising influence on scientific work, especially where the workers concerned approach the problem from the point of view of analysing the effects of a process, say, as structural engineers rather than as geologists, metrologists and so forth.

In this approach, a neat analytical function is usually selected, more or less arbitrarily, often out of consideration for the analytical simplicity of later steps in the analysis. For instance, in a paper concerned with the vibration of a beam-column under what the authors called an ‘earthquake-type’ load, a modal analysis of the deflection shapes was made on the basis of a stationary frequency distribution in which the intensity of the harmonics was normally distributed about zero, that is, the frequency spectrum was the error function. While the authors were quite aware of the arbitrariness of this hypothesis, they took the trouble to compute results in terms of the width of this error function.

In many instances, natural laws which represent only a partial truth in any particular case may provide an applied mathematician with an analytical function to represent a process which simplifies his algebra. Such idealistic approaches when applied at a particular stage in the study of some problem very often represent an advance. For instance, the use of Hookes law or the Normal Law of probability or the linear small-deflection equations in structural analysis can provide useful and meaningful results in the related problems, from which real behaviour can be seen as some sort of 'deviation'. However, especially where new phenomena arise, there is usually a point at which these 'approximations' become inadequate. Further, very often reality can be seen in terms of the co-existence of or opposition between these elementary forms; real behaviour can often be better understood by beginning with an identification of the elementary forms involved and their effects, then an understanding of how one type of behaviour may pass into another, rather than by considering the observed behaviour as quite indivisible. For instance, the behaviour of steel is usually approached by first understanding its elastic behaviour, then studying its plastic behaviour, then studying the significance of elastoplastic behaviour and later studying the effects of strain hardening, the fall-off in resistance towards the fracture point and strain-rate dependent behaviour. Alternatively. the effect of force and displacement loads may be considered in turn and may lead to an understanding of the unity of the structure and the so-called loading element which obeys a force-displacement law.

However, let us look at the origins of the almost universal agreement among earthquake engineers apart from those whose attention is confined solely to the use of digitised records, that an earthquake is, or may be validly considered as, a stationary stochastic acceleration of the ground.

Earthquakes endure for anything from 1 to 30 seconds. A single pulse, or cycle, way exceed one second, and in extreme cases the whole ground-motion may contain little more than a single cycle of the acceleration. That is, an earthquake is an eminently transient, or non-stationary process. Further, is structural response as a rule independent of the duration of the earthquake? Clearly, no.

While various modifications of the stationary stochastic process have found increasing application in earthquake-response studies, such as truncating the stationary process to a finite interval in time, the joining together in succession of say, three, different stationary processes, the use of non-random multiplicative envelopes, and the use of uniformly damped sinusoids or other modifications of the sinusoid as independent component functions, these techniques have not been very successful in facilitating either the computation of response or the modification of the process by any other form, because all but the simplest of transformations of a non-stationary stochastic process of this form are not 'closed' to that form. For instance, if a structure is subject to 5 seconds of each of 3 different stationary processes in succession, its response is a transient random function, the 3 components of which may be separable hut are not each stationary processes, nor confined to mutually exclusive time intervals. These techniques can be used to imitate a single, isolated record in much the same way a skilled draughtsman could do by hand.

The decision to consider earthquakes as stationary phenomena may be explicable in terms of the resulting analytical simplicity so long as one does not attempt to impose non-stationarity on the process in one of the above-mentioned, rather 'artificial' ways. However, the decision must derive in great part from a disturbing willingness to accept the validity of abstract concepts that bear no relation to observation, from the inertial effect of established theories, and even, in some spheres, especially the study of the fluctuation of economic parameters, a prejudice against the non-stationary process, that is, a process whose essence is in its development rather than in transient forms of its motion. It is interesting to note that, to the best of this author’s knowledge, the study of stochastic processes in economics is confined at present to stationary processes, this field being dominated by ‘statisticians’, while work which deals with the change and development of economic factors is based on what we could call ‘deterministic’ ideology and its exponents tend to reject probabilistic techniques except for the gathering of data. This phenomena of a split between ‘statisticians’ and ‘determinists’ is very general. Those earthquake engineers who prefer to use random function terminology and algebra more often tend to divorce their work from the actual physical processes which determine that random process, and consequently many earthquake engineers tend to see statistical techniques as essentially incapable of dealing with anything other than pen-on-paper records of the process as if they constituted the whole process. A determinist, on the other hand, faced with an enormous variety of forms of the same phenomenon, and an enormous number of factors involved in the generation of a process, without the aid of probabilistic concepts, is faced with a dilemma. Either he attempts to derive a prediction on a physical basis — but too many known and unknown influences are at work — or he confines his attention to records — but these are inadequate for such a variety of possibilities and remain unique, isolated events rather than members of a set of possible events ordered by probability distributions.

All workers of recent times whose specific area has been to study a particular natural phenomena have, fortunately, based their work on some field of observation of the process as it exists in nature. Thus, we will turn to look at the two approaches mentioned above, based principally on one or other of the two elements of the knowledge we have of a random process, the list of all past relevant events, and a knowledge of all the laws which relate to the generation of the event.

With regard to the first approach, we find that there is a ‘school’ in which the model of the process is the set of events which have in fact been recorded under one condition or another. This has the advantage that, if, for instance, we are talking about earthquakes, the model used in a given instance is guaranteed to bear some relation to something that was an earthquake, though the accelerogram is recognised to be a rather blurred image of the actual ground-motion, nevertheless, the best available. This mathematical model, which we could call empirical, is usually accompanied by a deterministic, usually digital-analogue computation of the response of a specified structure. This leaves the engineer with an passably accurate statement of what would result if such and such an earthquake had visited his proposed structure.

For instance, N. N. Ambrayses and his colleagues at Imperial College London have among other work on earthquakes, collected, zero-corrected and otherwise modified, and digitalised the 100 largest or most severe accelerograms available in the world. This momentous and valuable achievement they have applied by using highly refined digital analogue techniques to ‘subject’ structures in the design stage to each of these earthquakes in turn. The engineer is presumably to decide which of these records, and to what extent, represents earthquakes that could feasibly occur at his site? Are there aspects of that geology, or even of his structure which would tend to make one type of earthquake more likely than another? If 3 of the ‘top’ 50 earthquakes caused failure, does this warrant redesign? Is safety against the 1940 El Centro earthquake over-design or under-design? Just as design against the worst possible eventuality is becoming outmoded in many other spheres of structural design, so also must it in design against earthquakes, especially so since in most areas of the world the occurrence of a strong-motion earthquake is in itself an ‘extreme’.

Generally speaking, the use of a probabilistic concept is inevitable where the design process leads to a conflict or compromise: strength and safety versus cost, serviceability versus strength, lightness or flexibility versus toughness, and so on. The statistics of long-term variations, in terms of return periods of the magnitude applicable to a class of events and so on, provides a statistical technique compatible with any of the techniques for studying the nature of the individual events. However, we can see that many natural processes, and in particular, dynamic structural loads, cannot rationally be described by a single parameter. This is true also usually when we are concerned with analysing the effects of the process.

Thus, in the case of earthquakes, the need arises of first, investigation of, and then application of, geological knowledge, to, in effect, predict the nature of an earthquake applicable to a given site, corresponding also to other conditions.

Thus we find that, for instance, many Japanese workers have for a number of years been making considerable progress in this direction; notably, Kanhai, who has applied studies of the vibration of soil strata and the transmission of various types of waves by soils to predict stationary frequency distributions of earthquakes. Work has also been done in correlating observed frequency distributions with the related geology.

Further, it has been observed that the position of the fault relative to the 'recording site', the orientation, depth, extent, age and nature of the fault exert an influence on the resulting ground-motion, as does the geological nature of the earth mass transmitting the disturbance. The last is less important except for very near earthquakes since the large volume of material involved tends to cancel the effect of any non-homogenuities or physical properties peculiar to the area. Thus, the effect of this earth mass is much the same in one region as in another, other things being equal.

All these effects are, to a greater or lesser extent reproducible. Thus it is meaningful to talk of 'earthquakes on soft strata', 'near earthquakes', 'the earthquakes of such and such an area' and so on, as distinct phenomena each with their own characteristics. Further, strong-motion earthquakes also form a distinct class, distinguishable by nature as well as by magnitude, and to some extent by their origin.

Now, in areas where design of structures against earthquakes is a traditional, established part of the design process, that is to say, in parts of the world where the recommendations contained herein may be heard, one generally finds that a long history of earthquake phenomena has allowed geologists and others to identify and study particular faults, fault areas and earthquake-swarms. In addition, earthquakes are more likely to be taken into serious consideration, even if with no more justification, and in these cases, design of the foundations, or other considerations will generally necessitate a study of the underlying soil and bed-rock.

Thus, we see that where earthquakes are taken into account in design, the engineer may very often have access to a considerable mass of data relevant to at least the most likely and most severe earthquakes he may encounter at the site. Thus we see that it not unrealistic to talk of the application of the second class of knowledge which extends beyond bare records to the prediction of as yet unrecorded earthquakes.

The number of factors contributing to an earthquake ground-motion is very great. Any attempt to predict a ground-motion solely on the basis of site investigation is clearly idealistic, since the number of these factors will necessitate arbitrary selection of some of them. Further, the great variation in the nature of ground-motions recorded at the same site due to movements having the same origin shows that such an attempt is futile. This amounts to a demonstration that an earthquake ground-motion is essentially a random phenomenon.

However, the assertion that such consideration as we wish to give a random phenomenon must be based on both observation of past events, which represent some sort of summary of all the factors involved, and on the physical conditions appertaining to the records and the future event considered, is not negated by the assertion that the phenomenon is a random process.

A treatment of earthquakes which ignores the records as a source of data is not worth mentioning. However, an ill-attention to these records does influence some workers. Anyone would start a study of earthquakes by looking at these records (or at earthquake damage this depending mainly on opportunity). However, their closer study in their relation to the conditions of recording, and in their capacity as a standard against which results must be checked cannot be ignored.

In what way must these two opposed approaches be reconciled? If no relevant records existed we would be forced to adopt the second approach, since, in the case of earthquakes except for the detonation of explosives, which do not appear to produce a great amount of information useful to ground-motion studies, we do not have the opportunity of truly 'practising' from our study; that is, we cannot create earthquakes. This may not be applicable to some phenomena, such as the study of economic factors, mechanical vibration and shock.

In some cases, such as areas in which no strong-motion earthquakes have been recorded, but where further geological or historical information suggest that such may occur, the lack of records is dominant over the difficulty of speculation based on local conditions. A non-record in this respect tells only that the frequency of strong-motion earthquakes may be small, but the nature of an earthquake when it occurs is still unknown to us, since it is likely that a severe earthquake will be derived from a different source and will cause different response in the surrounding strata than weak earthquakes.

We suggest that prediction of the nature of a stochastic process begins from the best available records, for instance, strong-motion earthquake records preferably recorded at the same site, or possibly at geologically similar sites. An effort should then be made to reconcile these records with a set of geological conditions by simple but successively extensive mathematical models. — the direction of this growth of the mathematical models would appear to start with the structure itself, then to the soil immediately below the structure and successively outwards. This has been proved by the development of earthquake research over a period of time.

The mathematical models used to correlate with records must be non-stationary stochastic models, because the natural process itself is this. (I am talking specifically of earthquakes, but this applies to many other phenomena). If a deterministic model is used there is only an intuitive basis for comparison with the records, and no criterion for deciding whether a given record belongs to a hypothetical set of random functions associated with a certain mathematical model. Further, a stochastic model provides us with the opportunity of generating a variety of realisations of the random function all deemed to belong to the same random process in effect varying the unknown as well as the known factors. Clearly, any attempt to model a non-stationary process with a stationary model is doomed to failure.

At any given stage in the development of a mathematical model there may exist a purely idealistic element. For instance, we might try various analytical expressions to represent the motion of the bed-rock during an earthquake, although no observations of this way have been made, in order to model the effect of the strata of soil above bedrock. This is inevitable so long as knowledge of the process is advancing.

The next stage in the development of mathematical models is commenced when familiarity is obtained with a set of existing records and mathematical models which have been devised to represent them; that is, at the stage when existing records can be satisfactorily imitated. We should then vary our mathematical models to see if we can come to understand, in terms of our mathematical models, the origin of similarities and contrasts between different records in the context of the differences and similarities in the conditions under which they occurred.

Finally, we should vary our mathematical models in such a way as to fit new conditions peculiar to a case under consideration.

In modelling a particular stage in the generation of an earthquake ground-motion we must use what knowledge natural science gives us of this process to help us make an initial guess at a suitable mathematical model. Generation of the process mathematically and variation of the variable parameters of the model and comparison with the record leads to a modification of the model.

The mathematical model to be used by a designer would not be unique. That is, we would not aim to imitate nature to the extent of devising a model which produced randomly random functions of varying nature and intensity corresponding to the variation of all 'unpredictable' factors. However, we would aim to supply the designer with a technique for generating a small range of possible types of earthquake corresponding to the range of types of strong-motion earthquakes likely to occur at that site.

Such an investigation is obviously not the job of an individual design office, nor even of a single research group, but we are talking of a possible general direction for all research in these areas.

We do not recommend a compromise between imitation of records and models based on physical conceptions of the process, but rather a movement from one to the other which aims to create a new body of knowledge. This process has been developing for some time and has already produced some useful results. It is felt however that work in earthquakes and other non-stationary phenomena has been limited by an incomplete understanding of non-stationarity in stochastic processes.

In the following sub-section we wish to outline a few examples of the way in which the correlation spectrum may be used in the construction of mathematical models. The next part of the cycle correlation with existing records has not yet been begun by the author, although statistical imitation of the records has been developed, as the necessary techniques have only now just been collected together in this paper.

What has been said above regarding earthquakes applies, in the author's belief, in broad terms to many other phenomena. For instance, among structural loading actions we have: wind storms — variations in the speed of a gusty wind over a period of minutes especially during sudden increases in the mean wind speed or during changes of wind direction and cyclones; sea-waves during the peak of a storm; random vibrations due to impact and some mechanical sources. In other fields we have the fluctuation of economic parameters about their mean values, including wage levels, productivity and so on, especially during time of rapid economic change or development; in communications theory random electronic pulses and other transient phenomena, not to mention the random fluctuations treated in cybernetics. All these phenomena can be understood, with the aid of the correlation spectrum, in their non-stationarity.

Note that the particular choice of the sinusoid as the fundamental component function and the corresponding use of Fourier transforms, rather than the use of other orthonormal systems of functions is derived from the importance of oscillatory phenomena in very many natural Processes. In studying different phenomena we must be prepared to use different fundamental component functions.

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